SPICE Model for Piezoelectric Bender Generators

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***Abstract*— This paper presents the equivalent circuit model of PBG (Piezoelectric Bender Generator) along with its SPICE implementation. The model, made of lumped circuit elements, is valid under both parallel and series wired PBG operation modes. The model is validated by comparing the simulation results, from the transient analysis of the model in SPICE, with those obtained by using MATLAB®. An example using the proposed SPICE model is also presented.**

I. INTRODUCTION

Energy harvesting devices that can scavenge the normally lost energy from surroundings and convert it into usable form have gained much attention in recent years. A major source of the lost energy from surroundings is the ambient vibrations present around most machines and biological systems. Vibration based generators (VBG) have thus been explored extensively. VBGs are essentially second order mass-spring-damper systems that are made to vibrate by an external vibration source - indicated in terms of acceleration $\ddot{y}$ [1]. Three types of VBGs are usually reported in literature, i.e. electrostatic, electromagnetic and piezoelectric. However, the piezoelectric materials based VBGs are preferred as they efficiently convert mechanical strain to an electrical charge without any additional power. Among piezoelectric materials based VBGs, the piezoelectric bender generators (PBG), formed as cantilever beams such as bimorphs (two piezoceramic thin layers attached to a central metal shim) with a heavy proof mass placed on the free end (see Fig. 1), have received particular attention. Due to the design simplicity, high voltage and power outputs, the PBG has been used in this work.

A mathematical model of a PBG, taking into account its geometrical and physical parameters, as described in [2], allows better estimation of the amount of power that it is able to generate from a vibration source. Describing such a model in terms of circuit elements, as shown in Fig. 2, gives a designer the tool to study the PBG as an electrical circuit using the Kirchoff's Voltage and Current Laws. The SPICE implementation of this model, presented in this work, further allows a designer to simulate overall circuit (i.e. PBGs and the conditioning electronics) of the energy harvesting system. The SPICE implementation of PBG, presented in this work, is a better alternative to the simpler models (e.g. comprising of sinusoidal current generator in parallel with the bender capacitance) utilized in past [3]. With SPICE-like software, it would be easier to evaluate the performance of the overall circuit, from PBG to load, both in time and frequency domains. The optimum ways of transferring maximum power, from PBG towards the load, can be studied by directly simulating the overall circuit in SPICE.

This paper is organized as follows: Section II presents the electromechanical model of PBG. The model is similar to the one used in [2]. The SPICE implementation of the model is described in section III. Section IV provides the comparison of the SPICE simulation results with those obtained with MATLAB. An example of the proposed SPICE model is presented in section V and a summary of the work is given in the concluding section VI.

II. ANALYTICAL MODEL

In the electromechanical model of PBG, described in [2], the mechanical elements such as mass, spring and damper are represented by their electrical analogous quantities, i.e. equivalent inductance L_m, resistance R_b and capacitance C_k, respectively. Further, the piezoelectric coupling is modeled as an equivalent transformer - with transformer ratio *n,* as shown in Fig. 2. It is assumed that the piezoelectric layers are poled along the 3 axis, and the electrodes are placed on the top and bottom surfaces of the piezoceramic layers of the bimorph. Driving vibrations are assumed to be present along the 3 axis in such a manner that the piezoceramic material experiences a one-dimensional state of stress along the 1 axis. A uniform width of the PBG has been assumed. The proof mass is as-

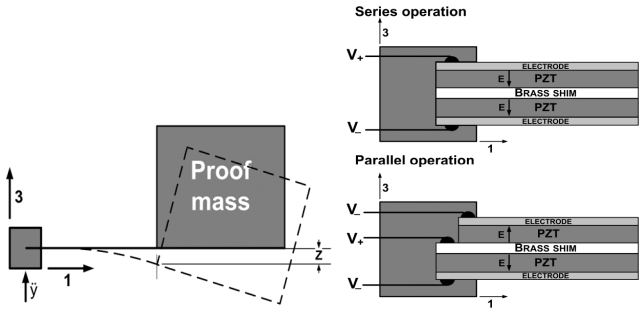

Figure 1. (*left*) Piezoelectric bender generator with a proof mass placed on the free end of the bender. (*right*) Piezoelectric bender generator wired for series and parallel operation mode.

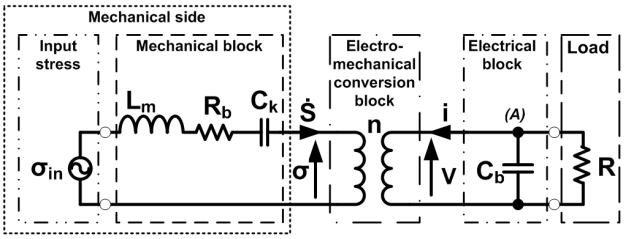

Figure 2. Piezoelectric bender generator electromechanical circuit model.

sumed to be a point mass located at center of mass. The piezoelectric losses are assumed to be negligible. Dielectric losses in piezoelectric materials can be explained using a complex dielectric constant [4], when computing the capacitance between the electrodes, C_b. This behavior can be modeled with a frequency-dependent resistance in parallel with the bender capacitance C_b [4]. The dielectric losses are higher if piezoelectric polymers are used in PBG. Dielectric loss factor in polymers are around 0.25, while in piezoceramic materials are in the range of 0.002-0.02. As piezoceramic based PBG is assumed in this work, the dielectric losses are negligible [5].

A. Mechanical side and electrical block equations

Considering the current as analogous to the strain rate $\dot{S}$, and voltage as analogous to the stress σ, and applying the Kirchoff's Voltage Law (KVL) across the mechanical block in Fig. 2, results in the following expression [2]:

$$\sigma_{in} - L_m\ddot{S} - R_b\dot{S} - S/C_k = nV\,. \tag{1}$$

Similarly, applying Kirchoff's Current Law (KCL) to node 'A' in Fig. 2, results in:

$$i = -C_b\dot{V} - V/R\,. \tag{2}$$

In (1), $\sigma_{in} = K_1 m\ddot{y}$ is the equivalent input stress, due to the input force (i.e. $F_{in} = m\ddot{y}$) exerted by the proof mass, m - when vibrations are applied at its base (see Fig. 1). Stress and force are related to through the geometrical constant K_1 [2]. The acceleration is defined as $\ddot{y} = a_{in}\sin(\omega_{in}t)$, where a_{in} is the acceleration magnitude and ω_{in} is the frequency of the acceleration in rad sec^{-1}. The equivalent inductance, $L_m = K_1K_2m$ which relates the second derivative of strain to stress, represents the inertia of the proof mass. The geometrical constant K_2 can be derived from the relation between the displacement, z, and strain [2]. The equivalent resistance, $R_b = K_1K_2b_m$ which relates stress to strain rate, takes into account the mechanical losses due to damping. In this equation, b_m is the damping coefficient. The equivalent capacitance, C_k, which represents the compliance, relates stress and strain, and it is simply equal to the inverse of the Young modulus, Y_p of the piezoelectric material.

In (2), the capacitance between the electrodes can be written as $C_b = \left(a^2wl_e\varepsilon\right)/\left(2t_p\right)$. In this relation, ε, l_e and w are the absolute piezoelectric dielectric constant, length and width of the electrode respectively and a is a constant which takes into account how the bimorph is wired (a = 1, if the bimorph is wired to operate in the series operation mode, or a = 2, if the bimorph is wired to operate in the parallel operation mode).

B. Piezoelectric Coupling Equations

The electromechanical conversion block, relating input stress on mechanical side to the voltage on electric block, is depicted by means of a transformer, as shown in Fig. 2. The equation for the transformer is given below:

$$\sigma = \left(\frac{-ad_{31}Y_p}{2t_p}\right)V = nV \tag{3}$$

where d_{31}, is the piezoelectric strain coefficient, and n is the equivalent voltage transformer ratio.

The current i flowing through the electrical block of the electromechanical bimorph model in Fig. 2, is generated as result of the mechanical stress applied to the bimorph under the conditions of zero electric field. The current i, can be related to strain rate $\dot{S}$ as:

$$i = \left(awl_e d_{31}Y_p\right)\dot{S}_1 = A_i\dot{S}_1\,. \tag{4}$$

The term between brackets is referred in the following sections as the equivalent current transformer ratio A_i.

III. SPICE IMPLEMENTATION

Modeling of the piezoelectric coupling is an important task when the analytical model, discussed in previous section, is implemented in SPICE. The equivalent transformer, used to depict piezoelectric coupling in Fig. 2, is different from the usual electrical transformer whose equations are related to the electromagnetic effects between the coils in the primary and secondary of the transformer. Usually, an electrical transformer is described as two inductances or inductively coupled conductors related by means of the transformer ratio.

An ideal electrical transformer can be described by means of the following equations

$$V_1 = \left(N_1/N_2\right)V_2 = n_V V_2 \tag{5}$$

$$I_2 = \left(N_2/N_1\right)I_1 = n_I I_1 = \left(1/n_V\right)I_1 \tag{6}$$

where, N_1, N_2, n_V and n_I are the number of turns in the primary and secondary, and voltage and current transformer ratio, respectively. It can be noted that for an ideal electrical transformer the current transformer ratio is the reciprocal of the voltage transformer ratio. However, the current transformer ratio A_i, obtained from (3) and (4), is not the reciprocal of the voltage transformer ratio n. This is an important difference between the electrical transformer and the equivalent transformer used to describe the piezoelectric coupling.

To describe and model the piezoelectric coupling by means of a transformer, only voltage transformer ratio n is not enough. The current transformer ratio A_i is also needed for a complete model of the piezoelectric coupling and hence to describe the equivalent transformer. According to (3) and (4),

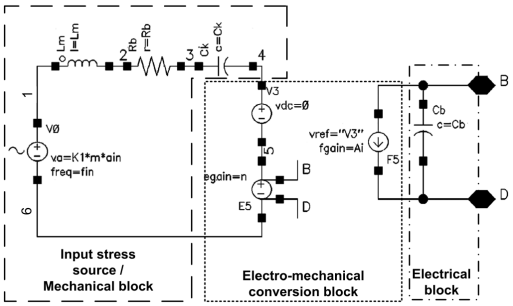

Figure 3. SPICE schematic subcircuit of the PBG electromechanical model.

controlled sources, available in SPICE, can be used to describe a complete model of the electromechanical piezoelectric coupling. A similar approach is used in [4] to model the electromechanical conversion in the SPICE model of the piezoelectric polymer.

In fact, (3) can also be modeled like a voltage controlled voltage source, (E5 in Fig. 3), with voltage across C_b (equivalent to the voltage across the secondary of the transformer) as the controlling parameter. The mechanical stress, σ (analogous to the voltage across the primary of the transformer) is the controlled analogous voltage in this case.

Similarly, (4) can be implemented in SPICE like a current controlled current source (F5 in Fig. 3). In this case, the controlling current is the analogous current flowing in the mechanical side i.e. the strain rate $\dot{S}$ and the controlled current is the current flowing in the secondary of the transformer i.e. i. The gain of E5 is the equivalent voltage transformer ratio n and the gain of F5 is equivalent current transformer ratio A_i. The mechanical stress, σ_{in}, is modeled in SPICE like a sinusoidal voltage source (V0 in Fig. 3).

An important characteristic of the proposed SPICE circuit implementation is its applicability to bimorph wired in both series and parallel operation modes (the differences between the two operating modes is due to the different wiring, as shown in Fig. 1). The parameters that depends on the operation mode are the values of the equivalent voltage and current transformer ratios, n and A_i respectively and the value of the capacitance C_b between the electrodes of the bimorph. This is due to the dependence of these parameters on the constant a - which defines the operation mode.

Fig. 3 shows the SPICE implementation of the electromechanical model of bimorph, discussed in previous section. The netlist for the equivalent circuit of Fig. 3 is given below:

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* Electromechanical Bimorph Equivalent Circuit
.subckt bimorph B D
V0 1 6 SINE(0 {K1*m*ain} {fin})
V3 4 5 0
E5 5 6 B D {n}
F5 D B V3 {Ai}
Lm 1 2 {Lm}
Rb 2 3 {Rb}
Ck 3 4 {Ck}
Cb B D {Cb}
.ends
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IV. Results and Discussion

To evaluate the model, the SPICE simulated plots of powers and voltages were compared with corresponding plots presented in [2]. For the comparison only, the physical parameter of the bimorph were assumed to be same as those used in [2]. The bimorph prototype used in [2], is made of two layers of PZT-5H with a brass center shim (from Piezo System, Inc). The bimorph has a length of 6.5 mm, width of 3.2 mm, thickness of each piezoceramic layers of 0.14 mm, and the thickness of the center metal shim is 0.1 mm. The proof mass, made of tungsten/nickel alloy (density – about 17.6 gm^{-3}), has length of 8.5 mm, width of 6.7 mm, and height of 7.7 mm. The bimorph is set to operate in parallel mode and a sinusoidal vibrations source of 2.5 ms^{-2} and frequency of 120 Hz (the bimorph was designed to resonate at 120 Hz), same as that used in [2], is assumed to excite the system. The sinusoidal stress source, σ_{in}, has a magnitude of 1.1727e6 Nm^{-1} and frequency of 120 Hz. A load resistance R, is assumed to be connected to the output of the bimorph model as in Fig. 2.

The average output power is computed using the formula,

$$P = V_{eff}^2 / R = \left(V_M / \sqrt{2}\right) / R = V_M / (2R) \qquad (7)$$

where V_{eff} is the rms voltage across the load resistance.

Fig. 4 compares the powers and rms voltages versus load resistance plots obtained both by means of MATLAB® and SPICE. The MATLAB® simulations, performed by solving the transfer function of the system [2], have been useful to validate the SPICE model of the bimorph. It is important to set the same solver method both in SPICE and MATLAB® (i.e. trapezoidal method) and the same simulation time step, to obtain similar results. As shown in Fig. 4, the powers and rms voltages versus load resistance agree well with each other. The plots shown in Fig. 4 are also in agreement with the corresponding plots presented in [2].

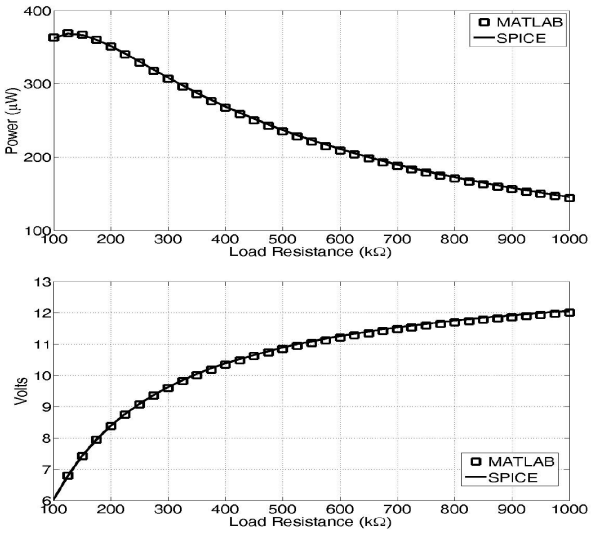

Figure 4. SPICE and MATLAB® powers and voltages vs load resistance in case of bimorph wired for the parallel operation mode.

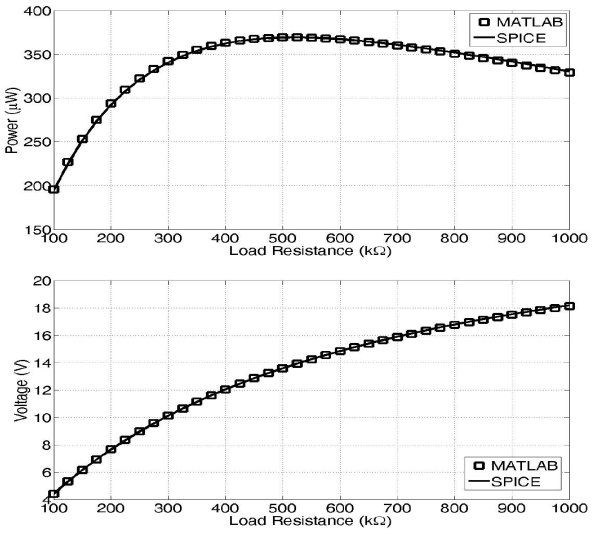

Figure 5. SPICE and MATLAB® powers and voltages vs load resistance in case of bimorph wired for the series operation mode.

As mentioned earlier, the SPICE implementation can also be used to evaluate the response of bimorph operating in series mode; only by changing the value of the constant a. The plots in Fig. 5 give the power and rms voltage of bimorph, operating in series mode, at various load conditions.

Comparing Fig. 4 and Fig. 5, it can be noted that the power is maximum at different load values. The bimorph operating in series mode shows maximum at an optimal load resistance greater than that of bimorph operating in parallel mode. For greater values of the optimal load resistance, the generated power varies more slowly in the case of the bimorph operating in series mode with respect to the power generated by the bimorph operating in parallel mode.

V. Application Example

To illustrate an application of the described model, a transient simulation of the SPICE bimorph model operating in parallel and series mode and connected to a diode bridge is given here. A 1N4148 diode model has been used for the bridge rectifier. The bimorph described in the earlier section is used. The other parameters of the SPICE bimorph model with the following values are assumed: R_b = 2.0622e6 Nsm^{-2}, C_k = 2e-11 m^2N^{-1}, L_m = 9.4501e4 kgm^{-1}, $C_{b,par}$ = 10e-9 F, n_{par} = 114286 $Nm^{-2}V^{-1}$, $A_{i,par}$ = – 665.6e-6 NmV^{-1}, $C_{b,ser}$ = 2.5e-9 F, n_{ser} = 57143 $Nm^{-2}V^{-1}$, $A_{i,ser}$ = – 332.8e-6 NmV^{-1}. A 3.3 μF storage capacitor, C_{st}, is connected to the output of the bridge rectifier. The sinusoidal stress source, σ_{in}, is the same described in the previous section.

Fig. 6 illustrates the schematic of the circuit used in SPICE to perform the transient analysis. Fig. 7 illustrates the plot of the rectified voltage curves for the two bimorph operation modes when the optimal load resistance is connected across the storage capacitor.

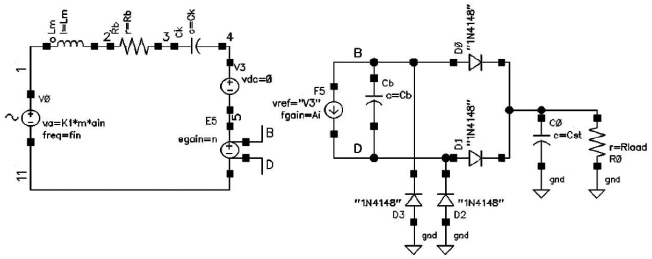

Figure 6. SPICE circuit for the example simulation.

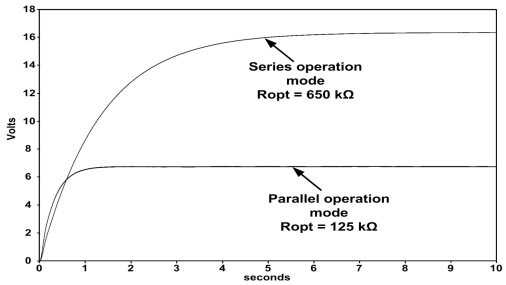

Figure 7. Simulated rectified voltage curves vs time, in case of optimal load resistance connected across the storage capacitor.

VI. Conclusions and Future Work

A SPICE implementation of a vibration-based piezoelectric bimorph generator for energy harvesting or scavenging applications has been presented. Comparing simulated voltages and powers, obtained with MATLAB® and SPICE, show a good agreement with each other and with the results reported in [2]. The MATLAB® and SPICE simulated powers and voltages of a bimorph, wired for the parallel and series operation modes, have been presented. The two operation modes have different generated power and optimal load resistance.

Future work will involve inclusion of various losses (dielectric, piezoelectric and viscoelastic) in the SPICE model so as to make it suitable for PBGs based on piezoelectric polymers. The future work will also involve designing an energy scavenging system that uses optimum parameters obtained from the SPICE model and a comparison of the results thereafter.

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